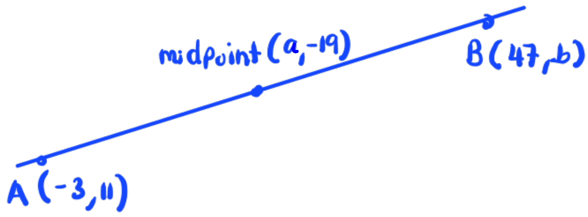
1 Point A has coordinates $(-3, 11)$
Point B has coordinates $(47, b)$
The midpoint of AB has coordinates $(a, -19)$

Find the value of a and the value of b.



$$a = \frac{(-3+47)}{2} = \frac{44}{2} = 22 \quad 1$$

$$-19 = \frac{(11+b)}{2}$$

$$-38 = 11 + b$$

$$b = -38 - 11$$

$$b = -49 \quad 1$$

$a =$ 22

$b =$ −49

(Total for Question 1 is 2 marks)

2 A curve has equation $y = f(x)$
There is only one maximum point on the curve.
The coordinates of this maximum point are (–3, 4)

Write down the coordinates of the maximum point on the curve with equation

(i) $y = f(x) - 6$

- value of x is not changing
- value of y translated down by 6 unit.

$(-3, 4-6) = (-3, -2)$

(-3 , -2) 1

(ii) $y = f(2x)$

- y-coordinate remains the same
- x-coordinate is divided by 2

(-1.5 , 4) 1

(Total for Question 2 is 2 marks)

3 The point A has coordinates $(5, -4)$
The point B has coordinates $(13, 1)$

(a) Work out the coordinates of the midpoint of AB.

midpoint AB : $\left(\frac{5+13}{2}, \frac{-4+1}{2}\right)$ 1

$= (9, -1.5)$ 1

(..........9.........., ...−1.5..........)
(2)

Line **L** has equation $y = 2 - 3x$

(b) Write down the gradient of line **L**.

$y = \underline{-3}x + 2$
$\uparrow$ m

..........−3 1..........
(1)

Line **L** has equation $y = 2 - 3x$

(c) Does the point with coordinates $(100, -302)$ lie on line **L**?
You must give a reason for your answer.

$y + 3x = 2$

LHS : $-302 + 3(100) = -2$. No. The coordinate does not lie on line L.

1

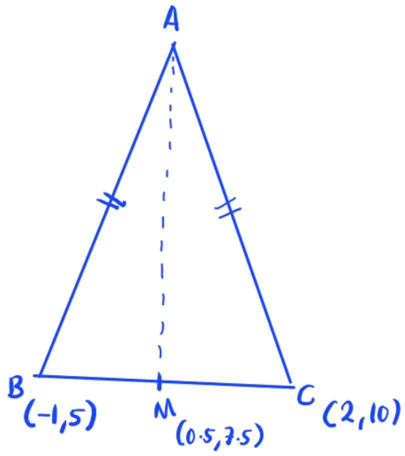
(1)

(Total for Question 3 is 4 marks)

4 *ABC* is an isosceles triangle with *AB* = *AC*.

B is the point with coordinates (–1, 5)
C is the point with coordinates (2, 10)
M is the midpoint of *BC*.

Find an equation of the line through the points *A* and *M*.
Give your answer in the form $py + qx = r$ where p, q and r are integers.



midpoint of BC $= \left(\frac{2+(-1)}{2}, \frac{10+5}{2}\right)$

$= (0.5, 7.5)$ (1)

gradient of line BC $: \frac{10-5}{2-(-1)}$

$= \frac{5}{3}$ (1)

gradient of line MA $= \frac{-1}{m_{BC}}$

$= -\frac{3}{5}$ (1)

Equation of line MA $= 7.5 = -\frac{3}{5}(0.5) + c$

$c = 7.5 + 0.3$

$= \frac{39}{5}$ (1)

$y = -\frac{3}{5}x + \frac{39}{5}$

$5y = -3x + 39$

$5y + 3x = 39$ (1)

$5y + 3x = 39$

....

(Total for Question 4 is 5 marks)

5 A rectangle *ABCD* is to be drawn on a centimetre grid such that

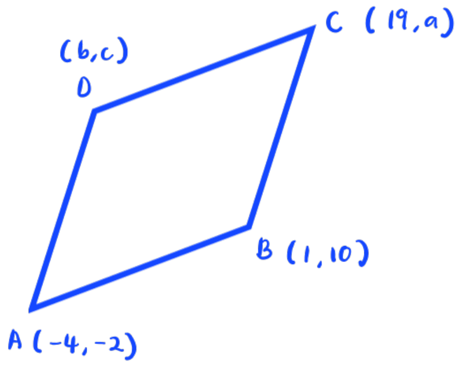
A has coordinates (–4, –2)
B has coordinates (1, 10)
C has coordinates (19, *a*)
D has coordinates (*b*, *c*)

(a) Work out the value of *a*, the value of *b* and the value of *c*.

Difference in x-axis between AB $= 1-(-4) = 5$

That means $b = 19-5$

$b = 14$ (1)



Gradient AB $= \dfrac{10-(-2)}{1-(-4)}$

$= \dfrac{12}{5}$ (1)

Gradient BC $= \dfrac{a-10}{19-1}$

$= \dfrac{a-10}{18}$

$\dfrac{12}{5} \times \dfrac{a-10}{18} = -1$

perpendicular lines :

$m_1 m_2 = -1$

$\dfrac{12(a-10)}{90} = -1$

$12a - 120 = -90$

$12a = 30$

$a = 2.5$ (1)

Difference in y-axis between AB $= 10-(-2)$

$= 12$

$c = 2.5 - 12 = -9.5$ (1)

$a =$ 2.5

$b =$ 14

$c =$ -9.5

(4)

(b) Calculate the perimeter, in centimetres, of rectangle *ABCD*.

$$AB = \sqrt{(1-(-4))^2 + (10-(-2))^2}$$
$$= \sqrt{5^2 + 12^2}$$
$$= 13 \quad (1)$$

$$BC = \sqrt{(19-1)^2 + (2.5-10)^2}$$
$$= 19.5 \quad (1)$$

$$\text{Perimeter} = 2(13) + 2(19.5)$$
$$= 65 \text{ cm} \quad (1)$$

65 cm

(3)

(Total for Question 5 is 7 marks)

6 Two circles, C_1 and C_2, are drawn on a centimetre grid, with a scale of 1 cm for 1 unit on each axis.

The centre of circle C_1 is at the point with coordinates (–1, 3) and the radius of C_1 is 13 cm.

The centre of circle C_2 is at the point with coordinates (7, 18) and the radius of C_2 is 6 cm.

(a) Work out the distance between the centre of C_1 and the centre of C_2

$(18-3)^2 + (7-(-1))^2 = 289$ (1)

distance : $\sqrt{289}$ (1)

$= 17$ (1)

17 .... cm

(3)

(b) Explain why circle C_1 intersects circle C_2

Total radii = 19 cm . Distance = 17 cm . They overlap by 2 cm . (1)

....

(1)

(Total for Question 6 is 4 marks)

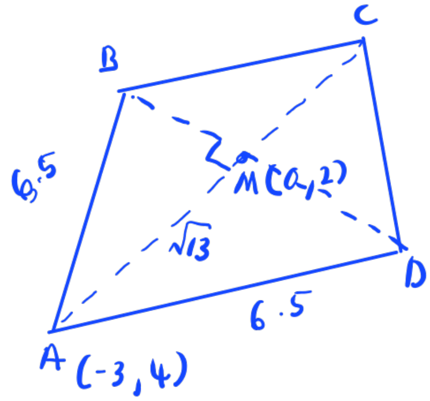
7 *ABCD* is a kite, with diagonals *AC* and *BD*, drawn on a centimetre square grid, with a scale of 1 cm for 1 unit on each axis.

A is the point with coordinates (−3, 4)

The diagonals of the kite intersect at the point *M* with coordinates (0, 2)

Given that *AB* = *AD* = 6.5 cm and the *x* coordinate of *B* is positive,

find the coordinates of the points *B* and *D*.



$m_{AM} = \frac{4-2}{-3} = -\frac{2}{3}$ 1

$m_{BD} = \frac{3}{2}$

Equation of line BD : $y - 2 = \frac{3}{2}x$

$y = \frac{3}{2}x + 2$ 1

$AM = \sqrt{(-3-0)^2 + (4-2)^2} = \sqrt{13}$

$BM = \sqrt{(x-0)^2 + (y-2)^2} = \sqrt{x^2 + (y-2)^2}$ 1

$AB^2 = AM^2 + BM^2$

$(6.5)^2 = 13 + x^2 + (y-2)^2$ 1

$\frac{117}{4} = x^2 + (y-2)^2$

$\frac{117}{4} = x^2 + \left(\frac{3}{2}x\right)^2$ 1

$\frac{117}{4} = \frac{13}{4}x^2$ 1

$x^2 = \frac{117}{13} = 9$

$x = \pm 3$

$x = 3, \ y = 6.5$

$x = -3, \ y = -2.5$

1

(3 , 6.5)

(-3 , -2.5)

(Total for Question 7 is 7 marks)

8 The line with equation $2y = x + 1$ intersects the curve with equation $3y^2 + 7y + 16 = x^2 - x$ at the points A and B

Find the coordinates of A and the coordinates of B
Show clear algebraic working.

$3y^2 + 7y + 16 = (2y-1)^2 - (2y-1)$ (1)

$3y^2 + 7y + 16 = 4y^2 - 4y + 1 - 2y + 1$

$3y^2 - 4y^2 + 7y + 6y + 16 - 2 = 0$

$-y^2 + 13y + 14 = 0$

$y^2 - 13y - 14 = 0$ (1)

$(y-14)(y+1) = 0$ (1)

$y = 14, \quad y = -1$

$x = 2(14) - 1 \quad , \quad x = 2(-1) - 1$
$= 27 \qquad\qquad = -3$ (1)

$(27, 14)$ and $(-3, -1)$
(1)

(27 , 14) and (-3 , -1)

(Total for Question 8 is 5 marks)

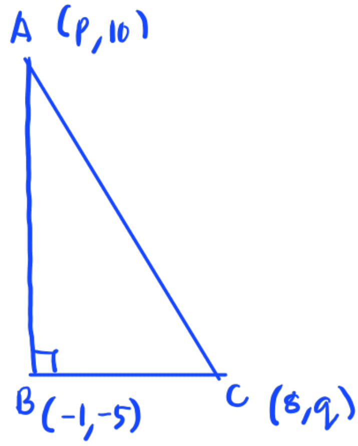
9 *ABC* is a triangle in which angle *ABC* = 90°

p and q are integers such that

the coordinates of A are $(p, 10)$
the coordinates of B are $(-1, -5)$
the coordinates of C are $(8, q)$

Given that the gradient of AC is $-\frac{6}{7}$

work out the value of p and the value of q



gradient AB : $\frac{10-(-5)}{p-(-1)} = \frac{15}{p+1}$ (1)

gradient BC : $\frac{q-(-5)}{8-(-1)} = \frac{q+5}{9}$

$\frac{\cancel{15}\,5}{p+1} \times \frac{q+5}{\cancel{9}\,3} = -1$ (1)

$5q+25 = -3p-3$ (1)

$5q+3p = -28$ — ①

$p = \frac{-28-5q}{3}$

gradient AC : $\frac{10-q}{p-8} = -\frac{6}{7}$

$70-7q = -6p+48$

$6p-7q = -22$ — ②

$\cancel{6}^{2}\left(\frac{-28-5q}{\cancel{3}\,1}\right) - 7q = -22$ (1)

$\therefore -56-10q-7q = -22$

$-17q = -22+56$

$-17q = 34$

$q = -2$

$$p = \frac{-28 - 5(-2)}{3}$$

$$= \frac{-28 + 10}{3}$$

$$= \frac{-18}{3}$$

$$= -6$$

$p =$ ......-6...... (1)

$q =$ ......-2......

(Total for Question 9 is 5 marks)

10 *ABCD* is a trapezium with *AB* parallel to *DC*

A is the point with coordinates (–4, 6)
B is the point with coordinates (2, 3)
D is the point with coordinates (–1, 8)

The trapezium has one line of symmetry.
The line of symmetry intersects *CD* at the point *E*

Work out the coordinates of the point *E*

midpoint AB : $\left(\frac{-4+2}{2}, \frac{6+3}{2}\right)$

$= (-1, 4.5)$ 1

gradient AB : $\frac{6-3}{-4-2} = -\frac{1}{2}$ 1

gradient of symmetry line = 2 1

DC: $y - 8 = -0.5(x - (-1))$

$y - 8 = -0.5x - 0.5$

$y = -0.5x + 7.5$ — 1 1

symmetry line : $y - 4.5 = 2(x - (-1))$

$y - 4.5 = 2x + 2$

$y = 2x + 6.5$ — 2 1

$2x + 6.5 = -0.5x + 7.5$

$2.5x = 1$

$x = \frac{1}{2.5} = 0.4$

$y = 2(0.4) + 6.5 = 7.3$

1

(0.4, 7.3)

(Total for Question 10 is 6 marks)

11 The points A and B are on a coordinate grid.

The coordinates of A are (6, 4)
The coordinates of B are (17, j) where j is a constant.

The midpoint of AB has coordinates (k, 15) where k is a constant.

Find the value of j and the value of k

$$k = \frac{6+17}{2} \quad (1)$$
$$= 11.5$$

$$\frac{4+j}{2} = 15 \quad (1)$$
$$j = 30 - 4$$
$$= 26 \quad (1)$$

$j =$ 26

$k =$ 11.5

(Total for Question 11 is 3 marks)

12 The diagram shows a triangle ABC where A, B and C represent the positions of three towns.

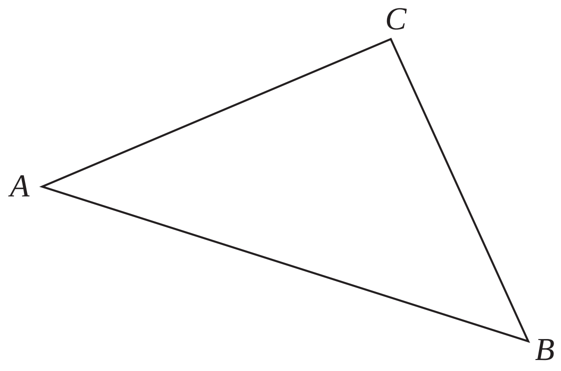


Diagram **NOT** accurately drawn

$$\overrightarrow{AB} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} \qquad \overrightarrow{BC} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$$

Pru travels directly from A to B and then directly from B to C

Yang travels directly from A to C

Given that the values for $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are in kilometres,

work out how much further Pru travels than Yang travels.
Give your answer in km, correct to one decimal place.

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$= \begin{pmatrix} 7 & -3 \\ -2 & +5 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ 1

distance $\overrightarrow{AC} = \sqrt{4^2 + 3^2}$

$= 5$ 1

distance $\overrightarrow{AB} = \sqrt{7^2 + (-2)^2}$

$= \sqrt{53}$

distance $\overrightarrow{BC} = \sqrt{(-3)^2 + 5^2}$ 1

$= \sqrt{34}$

total distance $= \sqrt{53} + \sqrt{34}$

$= 7.28\ldots + 5.83\ldots$

$= 13.11\ldots$ 1

difference $= 13.11 - 5$

$= 8.11$ 1

8.1 km

(Total for Question 12 is 5 marks)

13 Work out the coordinates of the points of intersection of

$$y - 2x = 1 \quad \text{and} \quad y^2 + xy = 7 \quad - (2)$$

Show clear algebraic working.

$y = 2x + 1 \quad - (1)$

substitute (1) into (2)

$(2x+1)^2 + (2x+1)x = 7$ (1)

$4x^2 + 4x + 1 + 2x^2 + x = 7$

$6x^2 + 5x - 6 = 0$ (1)

$(2x+3)(3x-2) = 0$ (1)

$x = -\frac{3}{2}$ and $x = \frac{2}{3}$

substitute x values into (1) :

$y = 2\left(-\frac{3}{2}\right) + 1$ and $y = 2\left(\frac{2}{3}\right) + 1$ (1)

$= -2$ and $\frac{7}{3}$

(1)

($-\frac{3}{2}$, -2)

($\frac{2}{3}$, $\frac{7}{3}$)

(Total for Question 13 is 5 marks)

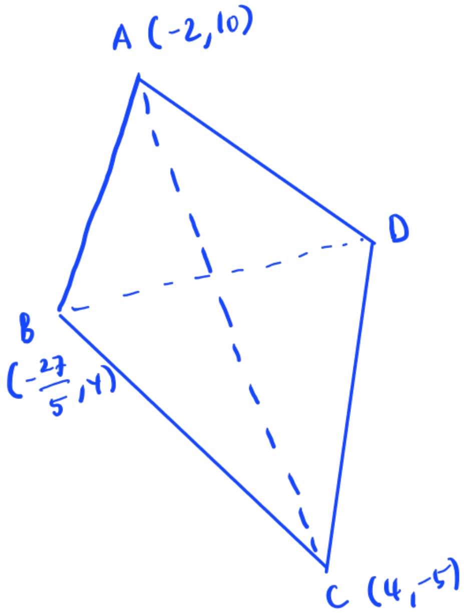
14 *ABCD* is a kite with *AB* = *AD* and *CB* = *CD*

A is the point with coordinates (–2, 10)

B is the point with coordinates $\left(-\frac{27}{5}, 4\right)$

C is the point with coordinates (4, –5)

Work out the coordinates of *D*



gradient AC : $\frac{-5-10}{4-(-2)} = \frac{-15}{6} = -\frac{5}{2}$ 1

equation of AC : $10 = -\frac{5}{2}(-2) + c$

$c = 10 - 5 = 5$

$\therefore \; y = -\frac{5}{2}x + 5$ 1

gradient BD : $\frac{2}{5}$

equation of BD : $4 = \frac{2}{5}\left(-\frac{27}{5}\right) + c$

$4 = -\frac{54}{25} + c$

$c = \frac{154}{25}$

$\therefore \; y = \frac{2}{5}x + \frac{154}{25}$ 1

$-\frac{5}{2}x + 5 = \frac{2}{5}x + \frac{154}{25}$ 1

$\frac{2}{5}x + \frac{5}{2}x = 5 - \frac{154}{25}$

$2.9x = -\frac{29}{25}$ 1

$x = -\frac{10}{25} = -\frac{2}{5}$, $y = -\frac{\cancel{5}}{\cancel{2}}\left(-\frac{\cancel{2}}{\cancel{5}}\right) + 5 = 6$

intersection between AC and BD is $\left(-\frac{2}{5}, 6\right)$

$$\left(-\frac{2}{5}, 6\right) = \left(\frac{-\frac{27}{5} + x_D}{2}, \frac{4 + y_D}{2}\right)$$

x_D : $-\frac{4}{5} + \frac{27}{5} = \frac{23}{5}$ 1

y_D : $12 - 4 = 8$

($\frac{23}{5}$, 8)

(Total for Question 14 is 6 marks)